EXAM 1 - MATH 341

Thursday, January 30, 2003 INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 10 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

- 1. (a) Let A, B be sets and $f : A \to B, g : B \to A$ be two functions such that $g \circ f = 1_A$. Show that f is one-to-one and that g is onto.
 - (b) Find an example of two sets A, B and two functions f, g, such that $g \circ f = 1_A$, f is not onto and g is not one-to-one.
- 2. In $\mathbb{R} \times \mathbb{R}$ consider the relation \sim , such that $(x_1, y_1) \sim (x_2, y_2)$ if and only if $3y_1 2x_1 = 3y_2 2x_2$. Determine whether \sim is an equivalence relation and, if so, describe its equivalence classes.
- 3. Show that n is prime if and only if in \mathbf{Z}_n , [r][s] = [0] always implies [r] = [0] or [s] = [0].
- 4. Find all the complex fourth roots of -1 i.
- 5. (a) Show that if A, B ∈ M(2, C) are invertible, then so is AB.
 (b) Find all matrices A with det(A) = 1 in M(2, Z₃).