EXAM 2 - MATH 341

Thursday, February 27, 2003
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 10 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Let G be a nonempty finite set closed under an associative operation such that both the left and the right cancellation laws hold. Show that G under this operation is a group.

2. Let G be a group, $a \in G$ and m, n relatively prime integers. Show that if $a^m = e$, then there exists an element $b \in G$, such that $a = b^n$.

3. Let G be a group and $a \in G$. Show that the centralizer $C(a)$ is a subgroup of G.

4. The **stochastic group** $\Sigma(2, \mathbb{R})$ consists of all those matrices in $\mathrm{GL}(2, \mathbb{R})$ whose column sums are 1. Show that this is in fact a subgroup of $\mathrm{GL}(2, \mathbb{R})$.

5. (a) Let $G = \langle a \rangle$ be a cyclic subgroup of order 20. Find all the elements $b \in G$ of order $|b| = 10$.

   (b) Let H and K be cyclic subgroups of an Abelian group G, with $|H| = 10$ and $|K| = 14$. Show that G contains a cyclic subgroup of order 70.