EXAM 4 - MATH 341

Thursday, April 24, 2003
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 10 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Let G be a group, $H \lhd G, \phi \in \mathrm{Aut}(G)$. Show that $\phi(H) \lhd G$.

   (b) Let G_1, G_2 be groups. Show that $Z(G_1 \times G_2) \cong Z(G_1) \times Z(G_2)$.

2. (a) Find the largest order of any element in $\mathbf{Z}_{21} \times \mathbf{Z}_{35}$.

   (b) Let G be an Abelian group and $\phi : G \to G$ a homomorphism such that $\phi(\phi(g)) = g$ for all $g \in G$. Show that $G \cong \phi(G) \times \mathrm{Kern}\phi$.

3. (a) Let p be a prime. Determine up to isomorphism all Abelian groups of order p^n that contain an element of order p^{n-2}.

   (b) Describe the positive integers n such that $\mathbf{Z}_n$ is up to isomorphism the only Abelian group of order n.

4. (a) Let R be a ring. If S and T are subrings of R, show that $S \cap T$ is also a subring of R.

   (b) Let R be a ring. Show that $(a+b)(a-b) = a^2 - b^2$ for all $a, b \in R$ if and only if R is a commutative ring.

5. (a) Give an example of a ring R and elements a, b and c in R such that $a \neq 0$, $ab = ac$, but $b \neq c$.

   (b) Find all the subdomains of $\mathbf{Z}$.