HOMEWORK 3 - MATH 341

DUE DATE: Tuesday, February 18

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Show that if G is a group and $a, b \in G$, then $|aba^{-1}| = |b|$.
   (b) Show that if G is a group and $a, b \in G$, then $|ab| = |ba|$.

2. (a) List all the cyclic subgroups of S_3. Does S_3 have a noncyclic proper subgroup?
   (b) List all the cyclic subgroups of D_4. Does D_4 have a noncyclic proper subgroup?

3. Let G be a group with no nontrivial proper subgroups.
   (a) Show that G must be cyclic.
   (b) What can you say about the order of G?

4. Let $\phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 4 & 1 & 6 & 7 & 2 & 5 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 4 & 1 & 2 & 7 & 8 & 5 & 3 \end{pmatrix}$.
   Calculate:
   (a) $\phi\tau$ and $\tau\phi$
   (b) $\phi^2\tau$ and $\phi\tau^2$
   (c) the inverses ϕ^{-1} and τ^{-1}
   (d) the orders $|\phi|$ and $|\tau|$.

5. (a) Show that if ρ and σ in S_n are disjoint cycles, and $\phi = \rho\sigma$, then $|\phi| = \text{lcm}(|\rho|, |\sigma|)$.
   (b) Show that an m-cycle is an even permutation if and only if m is odd.