HOMEWORK 4 - MATH 341

DUE DATE: Tuesday, February 25
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Show that for any $n \geq 3$, S_n is a non-Abelian group.
   (b) Show that every permutation $\rho \in S_n$ can be written as a product of 2-cycles of the form $(i \quad i+1)$, where $1 \leq i \leq n$.
2. Consider the regular tetrahedron.
   (a) Find all possible rotations of the regular tetrahedron.
   (b) The rotations of the regular tetrahedron correspond to elements of which known group?
3. (a) Find all cosets of the subgroup $5\mathbf{Z}$ in $\mathbf{Z}$.
   (b) Find the index of $\langle 10 \rangle$ in $\mathbf{Z}_{12}$ and the index of $\langle \mu_2 \rangle$ in S_3.
4. (a) Let H be a subgroup of a group G. For any $a, b \in G$, let $a \sim b$ if and only if $ab^{-1} \in H$. Show that the relation $\sim$ so defined is an equivalence relation on G, with equivalence classes the right cosets Ha of H.
   (b) Let H be a subgroup of a group G. Show for any $a \in G$ that $aH = H$ if and only if $a \in H$.
5. (a) Let G be a group with $|G| = p^2$, where p is a prime. Show that every proper subgroup of G is cyclic.
   (b) Let G be a group with $|G| = pq$, where p and q are primes. Show that every proper subgroup of G is cyclic.