HOMEWORK 6 - MATH 341 DUE DATE: Tuesday, March 18 INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

- 1. (a) Find all the normal subgroups in $GL(2, \mathbb{Z}_2)$, the general linear group of 2×2 matrices with entries from \mathbb{Z}_2 .
 - (b) Find all the normal subgroups in D_4 .
- 2. (a) Let $\phi : G \to G'$ be a homomorphism and $H' \lhd G'$. how that $H = \phi^{-1}(H') \lhd G$.
 - (b) Show that if $H \lhd G$ and $K \lhd G$, then $H \cap K \lhd G$.
- 3. (a) Show that if $H \triangleleft G$ and $K \triangleleft G$, then $HK \triangleleft G$.
 - (b) Let H and K be subgroups of a group G. Show that HK is a subgroup of G if and only if HK = KH.
- 4. Find the order of the indicated element in the indicated quotient group:
 - (a) $2 + \langle 6 \rangle$ in $\mathbf{Z}_{15} / \langle 6 \rangle$.
 - (b) $i\langle j\rangle$ in $Q_8/\langle j\rangle$.
- 5. (a) Let $\phi : G \to G'$ be an onto homomorphism with $\operatorname{Kern}(\phi) = K$, and let H' be a subgroup of G'. Show that there exists a subgroup H of G such that $K \subseteq H$ and $H/K \cong H'$.
 - (b) Let Z(G) be the center of a group G. Show that $Z(G) \lhd G$ and that, if G/Z(G) is cyclic, then G is Abelian.