HOMEWORK 7 - MATH 341 DUE DATE: Tuesday, March 25 INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

- 1. (a) Let $G = \langle a \rangle$ be a cyclic group of order 10. Describe explicitly the elements of Aut(G).
 - (b) Determine $\operatorname{Aut}(\mathbf{Z})$.
- 2. (a) Let G be an Abelian group. Show that the mapping $\phi: G \to G$ defined by letting $\phi(x) = x^{-1}$ for all $x \in G$ is an automorphism of G.
 - (b) Show that the mapping $\phi : S_3 \to S_3$ defined by letting $\phi(x) = x^{-1}$ for all $x \in S_3$ is not an automorphism of S_3 .
- 3. A subgroup H of a group G is called a *characteristic subgroup* of G if for all $\phi \in \operatorname{Aut}(G)$ we have $\phi(H) = H$.
 - (a) Show that if H is a characteristic subgroup of G, then $H \triangleleft G$.
 - (b) Show that if H is the only subgroup of G of order n, then H is a characteristic subgroup of G.
- 4. (a) Show that D_4 and $\mathbf{Z}_2 \times \mathbf{Z}_4$ are not isomorphic.
 - (b) In D_4 find a subgroup H such that $H \cong \mathbb{Z}_2 \times \mathbb{Z}_2$.
- 5. Let $H \triangleleft G_1$ and $K \triangleleft G_2$. Show that
 - (a) $H \times K$ is a subgroup of $G_1 \times G_2$
 - (b) $H \times K \lhd G_1 \times G_2$
 - (c) $(G_1 \times G_2)/(H \times K) \cong G_1/H \times G_2/K$