HOMEWORK 8 - MATH 341

DUE DATE: Tuesday, April 1

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Find the order of (ρ, i) in the group $S_3 \times Q_8$.
   (b) Find the order of $((2\ 3\ 4), 15)$ in $A_4 \times \mathbf{Z}_{18}$.
2. (a) Find the distinct cosets of $H = \langle (3,5) \rangle$ in the group $U(10) \times U(12)$.
   (b) Find the order of $(3,2) + \langle (6,8) \rangle$ in $(3\mathbf{Z} \times 2\mathbf{Z})/\langle (6,8) \rangle$.
3. (a) Explain why there are no nontrivial proper subgroups H and K in $\mathbf{Z}_8$ such that $\mathbf{Z}_8 = H \oplus K$.
   (b) Find nontrivial proper subgroups H and K in $U(12)$ such that $HK = U(12)$.
4. (a) Let H and K be subgroups of a group G such that $G = H \oplus K$, H is cyclic of order 6, and K is cyclic of order 15. Show that G is an Abelian group of order 90 that is not cyclic.
   (b) Let $G = H_1 \oplus \ldots \oplus H_n$, and let $x = h_1 + \ldots + h_n \in G$. Show that $|x| = \mathrm{lcm}(|h_1|, \ldots, |h_n|)$.
5. Let H and K be subgroups of an Abelian group G and let $\phi : G \to H$ be a homomorphism such that

   (1) $\phi(h) = h$ for all $h \in H$

   (2) $\mathrm{Kern}(\phi) = K$.

   Show that $G = H \oplus K$.