HOMEWORK 9 - MATH 341

DUE DATE: Tuesday, April 15

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Find up to isomorphism all Abelian groups of order 32 that have exactly two subgroups of order 4.

   (b) Let p be a prime. Determine how many Abelian groups there are of order p^5.

2. (a) Let p and q be distinct primes and G an Abelian group of order $|G| = n$, where both p and q divide n. Show that G contains a cyclic subgroup of order pq.

   (b) Let G_1 and G_2 be finite Abelian groups. Show that G_1 and G_2 have the same number of elements of order n for all n, if and only if $G_1 \cong G_2$.

3. (a) Let R be a ring. The **center** of R is defined as follows: $Z(R) = \{x \in R : xy = yx \text{ for all } y \in R\}$. Show that $Z(R)$ is a subring of R.

   (b) Find the center $Z(\mathbf{H})$ of the ring $\mathbf{H}$ of quaternions.

4. (a) A **Boolean ring** is a ring with the property that $a^2 = a$ for all $a \in R$. Show that a Boolean ring is a commutative ring with $2a = 0$ for all $a \in R$.

   (b) For any set X, let $P(X) = \{A : A \subseteq X\}$ be the powerset of X. For any A and B in $P(X)$ define $A + B = \{x : x \in A \cup B, x \notin A \cap B\}, A \cdot B = A \cap B$. Show that under these two operations $P(X)$ is a ring with unity that is a Boolean ring.

5. (a) Give an example of a commutative ring with no zero divisors that is not an integral domain.

   (b) Give an example of a ring with unity and no zero divisors that is not an integral domain.