EXAM 2 - MATH 490

Friday, March 21, 2003
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Read each problem very carefully before starting to solve it. Each question is worth 8 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Give the definition of a **topological space**.
   (b) Let X be an arbitrary set. Let $\mathcal{T}$ be the collection of all subsets of X whose complements are either finite or all of X. Then $(X, \mathcal{T})$ is a topological space.

2. (a) Give the definition of a **metrizable** topological space.
   (b) Prove that for each set X, the topological space $(X, 2^X)$ is metrizable.

3. (a) Give the definition of a **neighborhood space**.
   (b) Given a real number x, call a subset N of $\mathbb{R}$ a neighborhood of x if $y \geq x$ implies $y \in N$. Prove that this definition of neighborhood yields a neighborhood space. Describe the corresponding topological space.

4. (a) Let $(X, \mathcal{T})$ be a topological space and $A \subset X$. Give the definitions of the **closure, interior** and **boundary** of A.
   (b) In $\mathbb{R}^3$ with the usual topology, let A be the set of points $x = (x_1, x_2, x_3)$ such that $x_3 = 0$. Prove that $\mathrm{Int}(A) = \emptyset, \mathrm{Bdry}(A) = A$ and $\bar{A} = A$.

5. (a) Give the definition of a **Hausdorff space**.
   (b) Prove that a subspace of a Hausdorff space is a Hausdorff space.