HOMEWORK 5 - MATH 490

DUE DATE: Monday, March 31
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work.

GOOD LUCK!!

1. On the real line, prove that the set of non-zero numbers is not a connected set.

2. Let A and B be subsets of a topological space X. if A is connected, B is open and closed, and $A \cap B \neq \emptyset$, prove that $A \subset B$. (Hint: Assume $A \not\subset B$ and use the sets $P = A \cap B$ and $Q = A \cap C(B)$ to prove that A is not connected.)

3. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous. Prove that the image under f of each interval is either a single point or an interval.

4. Prove that a homeomorphism $f : [a, b] \to [a, b]$ carries end points into end points.

5. Prove that a polynomial of odd degree considered as a function from the reals to the reals has at least one real root.

6. Let $f : [a, b] \to \mathbb{R}$ be a continuous function from a closed interval into the reals. Let $U = f(u)$ and $V = f(v)$ be such that $U \leq f(x) \leq V$ for all $x \in [a, b]$. Prove that there is a w between u and v such that $f(w) \cdot (b - a) = \int_a^b f(t)dt$.

7. Prove that a nonempty connected subset of a topological space that is both open and closed is a component.

8. Let X be a topological space that has a finite number of components. Prove that each component of X is both open and closed.