HOMEWORK 9 - MATH 112
DUE DATE: Monday, April 19
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. One part of each problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Evaluate the improper integrals:

    (a) $\int_0^\infty \frac{5}{e^{2x}} dx$

    (b) $\int_{-\infty}^0 \frac{x}{x^2+1} dx$

2. Evaluate the improper integrals:

    (a) $\int_0^1 \frac{1}{x^2} dx$

    (b) $\int_3^5 \frac{1}{x^2\sqrt{x^2-9}} dx$

3. Use implicit differentiation to verify that the equation is a solution of the differential equation for any value of C.

    (a) $y^2 + 2xy - x^2 = C$, $\quad (x+y)y' - x + y = 0$

    (b) $x^2 - y^2 = C$, $\quad y^3 y'' + x^2 - y^2 = 0$

4. Verify that the general solution satisfies the differential equation. Then find the particular solution that satisfies the initial condition.

    (a) General Solution: $y = C_1 x + C_2 x^3$
       Differential Equation: $x^2 y'' - 3xy' + 3y = 0$
       Initial Condition: $y = 0$ and $y' = 4$ when $x = 2$.

    (b) General Solution: $y = (C_1 + C_2 x + \frac{1}{12}x^4)e^{2x}$
       Differential Equation: $y'' - 4y' + 4y = x^2 e^{2x}$
       Initial Condition: $y = 2$ and $y' = 1$ when $x = 0$.

5. Use integration to find the general solution of the differential equation:

    (a) $\frac{dy}{dx} = \frac{1}{2-7x}$

    (b) $\frac{dy}{dx} = x\sqrt{3x-5}$

    (c) $\frac{dy}{dx} = 11xe^{3x}$

6. Use separation of variables to find the general solution of the differential equation

    (a) $\frac{dy}{dx} = x^2 y$

    (b) $(1+y)\frac{dy}{dx} - 4x = 0$

    (c) $\frac{dy}{dx} = \frac{x^2+2}{3y^2}$

(d) $yy' - 2xe^x = 0$

7. Use the given initial condition to find the particular solution of the differential equation.

   (a) $yy' - e^x = 0, \quad y = 4$ when $x = 0$

   (b) $\sqrt{x} + \sqrt{y}y' = 0, \quad y = 4$ when $x = 1$

   (c) $\frac{dy}{dx} = x^2(1+y), \quad y = 3$ when $x = 0$.

8. Solve the differential equations

   (a) $\frac{dy}{dx} + 5y = 15$

   (b) $\frac{dy}{dx} + 3y = e^{-3x}$

   (c) $\frac{dy}{dx} + \frac{2y}{x} = 3x + 1$

   (d) $xy' + y = x^2 \ln x$