

## EXAM 2 - MATH 151

DUE DATE: Friday, February 20

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Read each problem very carefully before starting to solve it. Each question is worth 3 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the limits

- (a)  $\lim_{x \rightarrow 0} \frac{\sin^2 x}{3x^2}$
- (b)  $\lim_{x \rightarrow 0} \frac{\sin 6x}{\sin 8x}$
- (c)  $\lim_{h \rightarrow 0} \frac{1 - \cos 5h}{\cos 7h - 1}$
- (d)  $\lim_{t \rightarrow 0} \frac{t^2}{1 - \cos^2 t}$

2. Find a nonzero value for the constant  $k$  that makes

$$f(x) = \begin{cases} \frac{\tan kx}{3x}, & \text{if } x < 0 \\ 3x + k^2, & \text{if } x \geq 0 \end{cases}$$

continuous at  $x = 0$ .

3. Use the definition of the derivative to find  $f'(x)$  and then to find the equation of the tangent line to  $f(x) = \frac{3}{\sqrt{x-1}}$  at  $x = 2$ .

4. Find  $\frac{dy}{dx}$

- (a)  $f(x) = (x^7 - 4x^5 + 3)(x^{24} + 6x^3 - 17x)$
- (b)  $f(x) = \frac{\cos x + x^2}{4x - \tan x}$
- (c)  $f(x) = (4x^7 + \sec(4x^2))^5$

5. Find  $\frac{dy}{dx}$  by implicit differentiation

- (a)  $\cos(x^2 y^2) = x^2$
- (b)  $2x^3 = \frac{x+y}{x-y}$

6. Suppose that a spherical balloon is inflated so that, when its radius is 2 cm, the rate of change of its surface is  $32\pi$  cm<sup>2</sup>/min. Find the rate of change of its volume at the same time.<sup>1</sup>

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<sup>1</sup>Formulas for Volume and Surface of a Sphere:

$V = \frac{4}{3}\pi r^3$   
 $S = 4\pi r^2$