HOMEWORK 3 - MATH 151

DUE DATE: Monday, February 9
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. One part of each homework problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the discontinuities, if any
   (a) $f(x) = \sin(x^2 - 2)$
   (b) $f(x) = |\cos x|$
   (c) $f(x) = \frac{1}{1-2\sin x}$
2. Find the limits
   (a) $\lim_{x\to+\infty} \cos(\frac{1}{x})$
   (b) $\lim_{\theta\to 0} \frac{\sin 3\theta}{\theta}$
   (c) $\lim_{x\to 0^-} \frac{\sin x}{|x|}$
   (d) $\lim_{x\to 0^+} \frac{\sin x}{5\sqrt{x}}$
   (e) $\lim_{h\to 0} \frac{h}{\tan h}$
   (f) $\lim_{\theta\to 0} \frac{\theta}{\cos\theta}$
3. Find a nonzero value for the constant k that makes $f(x) = \begin{cases} \frac{\tan kx}{x}, & \text{if } x < 0 \\ 3x + 2k^2, & \text{if } x \geq 0 \end{cases}$ continuous at $x = 0$.
4. In each part find the limit by making the indicated substitution
   (a) $\lim_{x\to+\infty} x \sin\frac{1}{x}; \quad t = \frac{1}{x}$
   (b) $\lim_{x\to-\infty} x(1 - \cos\frac{1}{x}); \quad t = \frac{1}{x}$
   (c) $\lim_{x\to\pi} \frac{\pi - x}{\sin x}; \quad t = \pi - x$
5. Use the definition of the derivative to find $f'(x)$ and then to find the equation of the tangent line to $y = f(x)$ at $x = a$
   (a) $f(x) = 2x^3 + 1$ at $a = -1$
   (b) $f(x) = \sqrt{x-1}$ at $a = 8$
6. Find $\frac{dy}{dx}$
   (a) $y = -\frac{1}{3}(x^7 + 2x - 9)$
   (b) $y = -3x^{-8} + 2\sqrt{x}$
   (c) $f(x) = (3x^2 + 6)(2x - \frac{1}{4})$
   (d) $f(x) = (x^3 + 7x^2 - 8)(2x^{-3} + x^{-4})$

(e) $y = \frac{3x}{2x+1}$

(f) $y = \frac{2x-1}{x+3}$

(g) $y = (\frac{3x+2}{x})(x^{-5}+1)$

7. (a) Find a function $y = ax^2 + bx + c$ whose graph has an x-intercept of 1, a y-intercept of -2, and a tangent line with a slope of -1 at the y-intercept.

   (b) Find the x-coordinate of the point on the graph of $y = x^2$ where the tangent line is parallel to the secant line that cuts the curve at $x = -1$ and $x = 2$.

8. Let $f(x) = \begin{cases} x^2, & \text{if } x \leq 1 \\ \sqrt{x}, & \text{if } x > 1 \end{cases}$ Determine whether f is differentiable at $x = 1$. If so find the value of the derivative there.