

## HOMEWORK 5 - MATH 151

DUE DATE: Monday, March 8

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Read each problem very carefully before starting to solve it. One part of each homework problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. An aircraft is climbing at a  $30^\circ$  angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi/hr?
2. (a) Confirm that the stated formula is the local approximation at  $x_0 = 0$ 
  - i.  $(1+x)^{15} \approx 1 + 15x$
  - ii.  $\frac{1}{\sqrt{1-x}} \approx 1 + \frac{1}{2}x$(b) Confirm that the stated formula is the local linear approximation of  $f$  at  $x_0 = 1$ , where  $\Delta x = x - 1$ 
  - i.  $f(x) = x^4, (1 + \Delta x)^4 \approx 1 + 4\Delta x$
  - ii.  $f(x) = \frac{1}{2+x}, \frac{1}{3+\Delta x} \approx \frac{1}{3} - \frac{1}{9}\Delta x$
3. Use an appropriate local linear approximation to estimate the value of the given quantity
  - (a)  $\sqrt{65}$
  - (b)  $\sin 0.1$
4. The electrical resistance  $R$  of a certain wire is given by  $R = \frac{k}{r^2}$ , where  $k$  is a constant and  $r$  is the radius of the wire. Assuming that the radius  $r$  has a possible error of  $\pm 5\%$ , use differentials to estimate the percentage error in  $R$ . (Assume that  $k$  is exact.)
5. (a) Use the horizontal line test to determine whether  $f$  is one-to-one:
  - i.  $f(x) = |x|$
  - ii.  $f(x) = x^3$
  - iii.  $f(x) = \sqrt{x-1}$ .(b) Determine whether  $f$  is one-to-one by examining the sign of the first derivative:
  - i.  $f(x) = x^2 + 8x + 1$
  - ii.  $f(x) = 2x + \sin x$
  - iii.  $f(x) = \frac{x}{x+1}$ .
6. (a) Find a formula for  $f^{-1}(x)$ .
  - i.  $f(x) = 7x - 6$
  - ii.  $f(x) = \sqrt[3]{2x-1}$
  - iii.  $f(x) = \frac{x+1}{x-1}$

(b) Find a formula for  $f^{-1}(x)$  and state the domain of  $f^{-1}$

i.  $f(x) = (x + 2)^4, x \geq 0,$

ii.  $f(x) = \sqrt{x + 3}$

iii.  $f(x) = 3x^2 + 5x - 2, x \geq 0.$

7. (a) Find the exact values of the logarithms without using a calculator:

$$\log_2 16, \log_9 3, \log_{10} 0.00001, \ln \sqrt[2003]{e}.$$

(b) Expand or write as a single logarithm as appropriate:

i.  $\log \frac{\sqrt[3]{x+2}}{\cos 5x}$

ii.  $\ln \sqrt{\frac{x^2+1}{x^3+5}}$

iii.  $4 \log 2 - \log 3 + \log 16$

iv.  $\frac{1}{2} \ln(x + 1) - 3 \ln(\sin 2x) + 2$

8. (a) Solve the following equations:

i.  $\log_{10} x^2 + \log_{10} x = 30$

ii.  $\ln 4x - 3 \ln(x^2) = \ln 2$

iii.  $3e^{-2x} = 5$

iv.  $e^x - 2xe^x = 0$

v.  $e^{-2x} - 3e^{-x} = -2$

(b) Graph the following functions by using the three “critical” points and your knowledge of the graphs of exponentials and logarithms:

i.  $f(x) = 3 + e^{x-2}$

ii.  $f(x) = \left(\frac{1}{2}\right)^{x-1} - 1$

iii.  $f(x) = 3 \ln \sqrt[3]{x-1}$