

## HOMEWORK 6 - MATH 151

DUE DATE: Monday, March 15

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. One part of each homework problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the derivatives

(a)  $f(x) = \ln(\sin x)$

(b)  $f(x) = \ln(\ln x)$

(c)  $f(x) = \sqrt{1 + \ln^2 x}$

(d)  $f(x) = e^{-5x^2}$

(e)  $f(x) = \frac{e^x}{\ln x}$

(f)  $f(x) = e^{\sqrt{1+3x^2}}$

2. Use implicit or logarithmic differentiation as appropriate to compute the derivatives:

(a)  $y = \ln(x \tan y)$

(b)  $f(x) = \ln \frac{\cos x}{\sqrt{4-3x^2}}$

(c)  $f(x) = x\sqrt[5]{1+x^3}$

(d)  $f(x) = \frac{\sin x \cos x \tan^3 x}{\sqrt{x}}$

(e)  $f(x) = x^{\sin x}$

(f)  $f(x) = (x^2 + 3)^{\ln x}$

3. (a) Given that  $\theta = \sec^{-1} 2.6$  find the exact values of  $\sin \theta, \cos \theta, \tan \theta, \cot \theta$  and  $\csc \theta$ .

(b) Find the exact values of  $\cos^{-1}(\cos \frac{12\pi}{7}), \sin^{-1}(\sin \frac{5\pi}{7})$ .

4. Compute the derivatives

(a)  $f(x) = \tan^{-1}(x^2)$

(b)  $f(x) = \cot^{-1}(\sqrt{x})$

(c)  $f(x) = \sin^{-1}(\frac{1}{x})$

(d)  $f(x) = \ln(\cos^{-1} x)$

(e)  $f(x) = \sqrt{\cot^{-1} x}$

(f)  $f(x) = e^x \sec^{-1} x$

5. Use L'Hôpital's rule to find the limits:

- (a)  $\lim_{x \rightarrow \infty} \frac{3x-5}{2x+7}$
- (b)  $\lim_{x \rightarrow 1} \frac{x^2-1}{x^3-1}$
- (c)  $\lim_{x \rightarrow 1} \frac{\ln x}{x-1}$
- (d)  $\lim_{x \rightarrow 0} \frac{e^x-1}{\sin x}$
- (e)  $\lim_{t \rightarrow 0} \frac{te^t}{1-e^t}$

6. Use L'Hôpital's rule to find the limits:

- (a)  $\lim_{x \rightarrow \pi^+} \frac{\sin x}{x-\pi}$
- (b)  $\lim_{x \rightarrow 0^+} \frac{1-\ln x}{e^{\frac{1}{x}}}$
- (c)  $\lim_{x \rightarrow 0^+} \frac{\ln(\sin x)}{\ln(\tan x)}$
- (d)  $\lim_{x \rightarrow 0^+} (\tan x \ln x)$
- (e)  $\lim_{x \rightarrow \infty} (\cos(\frac{2}{x}))^{x^2}$

7. Study with respect to monotonicity and concavity the following functions:

- (a)  $f(x) = 4 - 3x - x^2$
- (b)  $f(x) = 3x^4 - 4x^3$
- (c)  $f(x) = \frac{x^2}{x^2+2}$
- (d)  $f(x) = xe^{x^2}$
- (e)  $f(x) = x^2 \ln x$

8. Identify all critical points and find the relative extrema using the first or the second derivative tests:

- (a)  $f(x) = x^4 - 2x^2 + 7$
- (b)  $f(x) = \frac{x^2-3}{x^2+1}$
- (c)  $f(x) = \ln(1+x^2)$
- (d)  $f(x) = x^2e^x$