EXAM 1 - MATH 216

Thursday, February 12, 2004

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Read each problem very carefully before starting to solve it. Each question is worth 2 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Give a combinatorial proof of Pascal's identity.

2. Give a formula and a proof for the number of r-combinations from a set with n elements when repetition of elements is allowed.

3. (a) Find an increasing subsequence of maximal length and a decreasing subsequence of maximal length in the sequence $22, 5, 7, 2, 23, 10, 15, 21, 3, 17$

   (b) Construct a sequence of 16 positive integers that has no increasing or decreasing subsequence of five terms.

4. Find the coefficient of

   (a) $x^5 y^8$ in $(x + y)^{13}$

   (b) x^9 in $(2 - x)^{19}$

5. (a) How many ways are there to distribute 12 indistinguishable balls into six distinguishable bins?

   (b) How many ways are there to distribute 15 distinguishable objects into five distinguishable boxes so that the boxes have one, two, three, four and five objects in them, respectively?