HOMEWORK 2 - MATH 216

DUE DATE: Tuesday, February 10
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

1. Every day a student randomly chooses a sandwich for lunch from a pile of wrapped sandwiches. If there are six kinds of sandwiches, how many different ways are there for the student to choose sandwiches for the seven days of a week if the order in which the sandwiches are chosen matters?

2. How many different ways are there to choose a dozen donuts from the 21 varieties at a donut shop?

3. How many solutions are there to the equation $x_1+x_2+x_3+x_4+x_5 = 21$ where $x_i, i = 1, 2, 3, 4, 5$ is a nonnegative integer such that

   (a) $x_i \geq 1$?
   (b) $x_i \geq 2$, for $i = 1, 2, 3, 4, 5$?
   (c) $0 \leq x_1 \leq 10$?
   (d) $0 \leq x_1 \leq 3, 1 \leq x_2 < 4$ and $x_3 \geq 15$?

4. How many ways are there to distribute 12 indistinguishable balls into six distinguishable bins?

5. How many ways are there to distribute 15 distinguishable objects into five distinguishable boxes so that the boxes have one, two, three, four and five objects in them, respectively?

6. How many different bit strings can be formed by using six 1's and eight 0's?

7. What is the probability that the sum of the numbers on two dice is even when they are rolled?

8. What is the probability that a coin lands heads up six times in a row?

9. What is the probability that a 5-card poker hand contains at least one ace?

10. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 7?

11. What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

    (a) The permutation consists of the letters in reverse alphabetic order.
    (b) z is the first letter of the permutation.
    (c) z precedes a in the permutation.

(d) a immediately precedes z in the permutation.

(e) a immediately precedes m, which immediately precedes z in the permutation.

(f) m,n and o are in their original places in the permutation.

12. Suppose that E and F are events such that $p(E) = 0.7$ and $p(F) = 0.5$. Show that $p(E \cup F) \geq 0.7$ and that $p(E \cap F) \geq 0.2$.

13. (a) What is the probability that two people were born during the same month of the year?

    (b) What is the probability that in a group of n people there are at least two born in the same month of the year?

    (c) How many people are needed to make the probability greater than $\frac{1}{2}$ that there at least 2 people born in the same month of the year?

14. Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p.

    (a) the probability of no failures

    (b) the probability of at least one failure

    (c) the probability of at most one failure

    (d) the probability of at least two failures.

15. What is the expected sum of the numbers that appear on two dice, each biased so that a 3 comes up twice as often as each other number?

16. The final exam of a discrete mathematics class consists of 50 true/false questions, each worth two points and 25 multiple choice questions each worth four points. The probability that Linda answers a true/false question correctly is 0.9 and the probability that she answers a multiple-choice question correctly is 0.8. What is her expected score on the final?

17. Estimate the expected number of integers with 1000 digits that need to be selected at random to find a prime, if the probability a number with a 1000 digits is prime is approximately $\frac{1}{2302}$.

18. Let X_n be the random variable that contains the difference in the number of tails and the number of heads when n coins are flipped.

    (a) What is the expected value of X_n?

    (b) What is the variance of X_n?