

HOMEWORK 1 - MATH 112

DUE DATE: Monday, January 16

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. One part of each problem will be chosen at random and graded. Each question is worth 1 point. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Let $f(x) = \frac{2}{x}$ and $g(x) = \sqrt{x-1}$. Find a formula for the composite $(g \circ f)(x)$. Then find the domain of $g \circ f$.
2. Find the inverse function of $f(x) = \frac{2x-1}{3+x}$.
3. (a) Let $f(x) = \begin{cases} \frac{1}{3}x - 2, & \text{if } x \leq 3 \\ -2x + 5, & \text{if } x > 3 \end{cases}$
Find $\lim_{x \rightarrow 3^-} f(x)$ and $\lim_{x \rightarrow 3^+} f(x)$. Does $\lim_{x \rightarrow 3} f(x)$ exist?
(b) Let $f(x) = \begin{cases} x^2 - 5x + 1, & \text{if } x < -1 \\ 2x + 7, & \text{if } x > -1 \end{cases}$
Find $\lim_{x \rightarrow -1^-} f(x)$ and $\lim_{x \rightarrow -1^+} f(x)$. Does $\lim_{x \rightarrow -1} f(x)$ exist?
4. Find the following limits:
 - (a) $\lim_{x \rightarrow 2} (2x - 3)$
 - (b) $\lim_{x \rightarrow -2} \frac{x^2-1}{2x}$
 - (c) $\lim_{x \rightarrow 4} \sqrt{x+4}$
5. Find the following limits:
 - (a) $\lim_{x \rightarrow -1} \frac{2x^2-x-3}{x+1}$
 - (b) $\lim_{x \rightarrow 2} \frac{2-x}{x^2-4}$
6. Let $f(x) = \begin{cases} 3+x, & \text{if } x \leq 2 \\ x^2+1, & \text{if } x > 2 \end{cases}$. Test f for continuity at $x = 2$ **in detail**.
7. Find the value of the constant a so that the function
$$f(x) = \begin{cases} x^3 + 1, & \text{if } x \leq 2 \\ ax^2 + 7, & \text{if } x > 2 \end{cases}$$
be continuous at $x = 2$.
8. Look at Example 2 on page 63 in your book. Discuss the continuity of the functions $f(x) = \frac{x^2-9}{x+3}$ and $g(x) = \frac{1}{x+7}$.