PRACTICE EXAM 1 - MATH 112
DATE: Friday, January 27
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 3 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the following limits:

   (a) $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1}$ (1 point)

   (b) $\lim_{x \to 0} \frac{\sqrt{x+3} - \sqrt{3}}{x}$ (2 points)

2. (a) Give the *formal* (not geometric) definition of a function $y = f(x)$ being continuous at $x = a$. (1 point)

   (b) Consider the function $f(x) = \begin{cases} \frac{1}{2}x + 1, & \text{if } x < 2 \\ 3 - x, & \text{if } x > 2 \end{cases}$ Does $\lim_{x \to 2} f(x)$ exist? Is $f(x)$ continuous at $x = 2$? Explain your answers *formally*. (1 point)

   (c) Determine a so that the function $f(x) = \begin{cases} 3x + 5 & \text{if } x \leq -1 \\ ax - 9, & \text{if } x > -1 \end{cases}$ be continuous at $x = -1$. (1 point)

3. (a) The limit definition of the derivative says that $f'(a) =?$ (1 point)

   (b) Use the limit definition of the derivative to compute the derivative of the function $f(x) = \sqrt{x+2}$ at the point $x = 2$. (2 points)

4. (a) Find the derivative of $f(x) = 3x^5 - 4x^3 + x^2 - 10$. (1 point)

   (b) Find the derivative of $g(x) = \frac{5}{x^3} + \frac{8}{\sqrt[8]{x^7}}$. (1 point)

   (c) Find the equation of the tangent line to the graph of $y = g(x)$ at $x = 1$.

5. Find the derivatives of the functions:

   (a) $f(x) = (x^2 - 4x + 3)(7 - x^2 - 3x^7)$ (1 point)

   (b) $g(x) = \frac{x^2 + 4x - 8}{5 - x^3}$ (2 points)

6. Find the points, if any, at which the graph of $f(x) = \frac{x^4 + 3}{x^2 + 1}$ has a horizontal tangent line. (3 points)