TEST 10 - MATH 140

DATE: Friday, March 24

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Sketch the graph of the **function** $y = \cos x$ **that has an inverse**. (1 point)
   (b) Sketch the graph of the inverse $y = \cos^{-1} x$ of the function whose graph you sketched in the previous part. (2 points)
   (c) Find the exact value of the expression $\cos^{-1}(-\frac{1}{2})$ and the exact value of the expression $\cos^{-1}(\cos\frac{19\pi}{6})$. (1 point)
   (d) Find the exact value of the expression $\sin^{-1}(-\frac{\sqrt{2}}{2})$ and the exact value of the expression $\sin^{-1}(\sin\frac{4\pi}{3})$. (1 point)

2. (a) Find the exact value of the expression $\csc(\cos^{-1}(-\frac{\sqrt{3}}{2}))$. (1 point)
   (b) Find the exact value of the expression $\sec(\sin^{-1}(\frac{2\sqrt{5}}{5}))$. (2 points)
   (c) Find the exact value of the expression $\sin(\tan^{-1}(\frac{1}{2}))$. (2 points)

3. (a) Establish the identity $\frac{\cot\theta}{1-\tan\theta} + \frac{\tan\theta}{1-\cot\theta} = 1 + \tan\theta + \cot\theta$. (2.5 points)
   (b) Establish the identity $\frac{\cos^2\theta - \sin^2\theta}{1-\tan^2\theta} = \cos^2\theta$. (2.5 points)

4. (a) Show that $\sin(\cos^{-1}(v)) = \sqrt{1-v^2}$. (2 points)
   (b) Find the exact value of $\cos\frac{19\pi}{12}$. (1 point)
   (c) Establish the identity $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$. (2 points)

5. (a) Suppose $\tan\alpha = \frac{5}{12}, \pi < \alpha < \frac{3\pi}{2}$. Find $\sin\alpha$ and $\cos\alpha$. (1 point)
   (b) Suppose $\sin\beta = -\frac{4}{5}, -\frac{\pi}{2} < \beta < 0$. Find $\sin\beta$ and $\cos\beta$. (1 point)
   (c) Find the exact value of the expression $\cos(\alpha-\beta)$, where α and β are the angles in the previous two parts. (1 point)
   (d) Find the exact value of the expression $\cos(\tan^{-1}\frac{4}{3} + \cos^{-1}\frac{12}{13})$. (2 points)