TEST 12 - MATH 140

DATE: Friday, April 7

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Solve the following trigonometric equations:
   (a) $\tan\theta = \sqrt{3}$. (2 points)
   (b) $\cos(2\theta + \frac{\pi}{4}) = \frac{\sqrt{3}}{2}, 0 \leq \theta < 2\pi$. (3 points)
2. Solve each equation in the interval $0 \leq \theta < 2\pi$.
   (a) $\cos(2\theta) + 6\sin^2\theta = 4$ (3 points)
   (b) $\sec\theta = \tan\theta + \cot\theta$. (2 points)
3. To measure the height of a building, two sightings are taken a distance of 50 feet apart (on the same side of the building). The first angle of elevation is 40° and the second is 32°.
   (a) Make a sketch of the geometry, denoting by h the height of the building and by x the distance of the closest sighting from the building. (1 point)
   (b) Based on right triangle trigonometry **only**, express h in terms of x in two different ways. (2 points)
   (c) Use the two expressions that you found in part (b) to find an expression calculating x. (1.5 points)
   (d) Use the expression obtained in part (c) to compute the height h. (0.5 points)
4. An aircraft is spotted by two observers who are 1000 feet apart. As the airplane passes over the line joining them, each observer takes a sighting of the angle of elevation to the plane. The first observer's angle is 47° and the second observer's angle is 35°.
   (a) Make a sketch of the geometry of the problem indicating the position of each observer and denoting by h the height of the airplane, by a the distance of the airplane from the first observer and by b the distance of the airplane from the second observer. (1 point)
   (b) Provide an expression for computing a involving only known quantities. (2 points)
   (c) Use the expression obtained in part (b) to compute the height h of the airplane. (2 points)
5. Use your Half-Angle Formulas and the Law of Cosines to show that in any triangle the following formula holds:
$$\cos(\frac{\gamma}{2}) = \sqrt{\frac{(a+b+c)(a+b-c)}{4ab}}. \quad \text{(5points)}$$