

HOMEWORK 1 - MATH 216

DUE DATE: When 1.4 has been covered.

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Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

- 1.1 The social security number of a person is a sequence of nine digits that are not necessarily distinct. Find the cardinality of the set of all social security numbers.
- 1.4 Find the number of even numbers between 0 and 100 and the number of even numbers with distinct digits between 0 and 100.
- 1.9 There are n married couples in a group. Find the number of ways of selecting a woman and a man who is not her husband from this group.
- 1.16 Find the value of the positive integer n if (a) $P(n, 2) = 30$, (b) $P(n, 3) = 24P(n, 2)$, (c) $10P(n, 2) = P(3n - 1, 2) + 40$.
- 1.21 For station wagons, five sedans and six vans are to be parked in a row of 15 parking spots. Find the number of ways of parking these vehicles such that (a) the station wagons are parked at the beginning, then the sedans and then the vans. (b) vehicles of the same type are parked en bloc.
- 1.25 Six girls and six boys are to be assigned to stand around a circular fountain. Find the number of such assignments if on either side of a boy there is a girl and on either side of a girl there is a boy.
- 1.30 Find the number of ways in which the complete collection of letters that form the word MISSISSIPPI can be arranged such that (a) there is no restriction on the location of the letters, (b) all the S 's stay together.
- 1.35 There are 4 women and 9 men in the mathematics faculty of a college. Find the number of ways of forming a hiring committee consisting of 2 women and 3 men from the department.
- 1.40 Find the number of bytes that can be formed using exactly 6 zeros.
- 1.43 Prove the following identity using a combinatorial argument:

$$\begin{aligned} C(n, r) &= C(r, r)C(n - r, 0) + C(r, r - 1)C(n - r, 1) \\ &\quad + C(r, r - 2)C(n - r, 2) + \cdots \\ &\quad + C(r, 1)C(n - r, r - 1) + C(r, 0)C(n - r, r). \end{aligned}$$

- 1.47 Let X be the set of all words of length 10 in which the letter P appears 2 times, Q appears 3 times and R appears 4 times. Find the cardinality of X .

- 1.50 Find the coefficient of $a^2b^3c^3d^4$ in the expansion of $(a + b + c + d)^{12}$ and of $(2a - 3b + 2c - d)^{12}$.
- 1.51 Use Pascal's triangle and list the coefficients of the terms which appear in the expansion of $(x + y)^n$ for $n = 4, 7$ and 11 .
- 1.57 There are 18 students in a class. Find the number of ways of partitioning the class into (a) 4 groups of equal strength and a minority group, (b) 2 groups of 5 students, 1 group of 4 students, and 2 groups of 2 students (c) 1 group of 7 students, 1 group of 6 students and 1 group of 5 students.
- 1.62 Find the number of ways of forming a team of 15 students from a large university to represent freshmen, sophomores, juniors, seniors and graduate students such that the team has (a) at least one from each group, (b) at least two from each group, (c) at least two graduate students.
- 1.66 When a die is rolled, one of the first six positive integers is obtained. Suppose that the die is rolled five times and the sum of the five integers thus obtained is added. The five throws constitute a trial. Find the number of possible trials such that the sum is at most 12.
- 1.68 Find the number of solutions in nonnegative integers of the equation $x_1 + x_2 + x_3 + 3x_4 = 7$.