HOMEWORK 3 - MATH 216
DUE DATE: When Chapter 2 has been covered.
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

2.1 Find the ordinary generating functions of the following sequences:
   (a) $1, 1, 1, 1, 0, 0, 0, \ldots$
   (b) $0, 0, 0, 0, 1, 1, 1, \ldots$
   (c) $1, 1, 1, 1, \ldots$
   (d) $1, -1, 1, -1, \ldots$

2.2 Find the ordinary generating functions for the following sequences: (a) $1, 2, 3, 4, \ldots$ (b) $1, -2, 3, -4, \ldots$.

2.3 Find the sequence corresponding to the following ordinary generating functions: (a) $(2 + x)^4$ (b) $x^2 + e^x$ (c) $\frac{x^3}{1-x}$.

2.6 Find the coefficient of x^{23} in $(x^3 + x^4 + \cdots)^5$.

2.8 Find the ordinary generating function associated with the problem of finding the number of solutions in nonnegative integers of the equation $3a + 2b + 4c + 2d = r$.

2.11 If three distinct dice are thrown, find the number of ways of getting a total of 13.

2.14 Find the ordinary generating function associated with the combinatorial problem of choosing 9 marbles from a bag that has 3 identical red marbles, 4 identical blue marbles and 5 identical green marbles such that in every choice all colors are represented and no color has absolute majority.

2.17 There are 10 identical gift boxes. Each box has to be wrapped with either red or blue or green or yellow wrapping paper. The available red paper can be used to wrap at most 2 boxes and the available blue paper can be used to wrap at most 3 boxes. Write down the ordinary generating function associated with the problem of finding the number of ways of wrapping these 10 boxes.

2.20 A participant in a contest is rated on a scale of 1 to 6 by each of the 4 judges. To be a finalist a participant has to score at least 22. Find the number of ways the judges can rate a participant so that she can be a finalist.

2.26 Find the ordinary generating functions for the sequences: (a) $a_r = k^r, k$ a constant (b) $b_r = rk^r$ (c) $c_r = k + 2k^2 + 3k^3 + \cdots + rk^r$.

2.29 Let $X = \{1, 2, \ldots, n\}$. Find the number of subsets of X such that each subset has r elements and no two elements in a subset are consecutive integers.

2.33 Show that every nonnegative integer can be written uniquely in a binary form.

2.34 Find the exponential generating function associated with the following sequences:

    (a) $1, 1, 1, 1, 0, 0, 0, \ldots$,

    (b) $0, 0, 0, 0, 1, 1, 1, \ldots$,

    (c) $1, 2, 2^2, 2^3, \ldots$

    (d) $1, 1, 2 \cdot 2, 3 \cdot 2^2, 4 \cdot 2^3, \ldots$.

2.36 Let $X = \{A, B, C, D\}$. Using exponential generating functions obtain (a) the number of r-permutations that can be formed using these four letters such that in each permutation there is at least one A, at least one B and at least one C and (b) the number of r-permutations that can be formed such that in each permutation there is an even number of A's and an odd number of B's.

2.38 (a) Find the number of ways the headquarters of a company can allocate nine new identical computers to four distinct branch offices so that each office gets at least one new computer. (b) Find the number of ways the headquarters of a company can allocate nine new employees to four distinct branch offices so that each office gets at least one new employee.

2.40 Find the number of nine-digit sequences that can be formed using the digits 0,1,2 and 3 such that (a) each sequence has an even number of 0's (b) each sequence has an odd number of 0's (c) each sequence has an even number of 0's and an odd number of 1's (d) the total number of 0's and 1's is odd (e) no digit appears exactly twice.

2.41 Obtain the appropriate generating function associated with the combinatorial problem of finding the number of codewords of length r from an alphabet consisting of five distinct letters such that in each codeword every letter of the alphabet appears at least once and the first letter appears an even number of times.