HOMEWORK 4 - MATH 216
DUE DATE: When Chapter 3 has been covered.
INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. A few randomly selected problems will be graded for a total of 10 points. It is necessary to show your work.

GOOD LUCK!!

3.1 Suppose that there are n lines in a plane such that no two lines are parallel and no three lines are concurrent dividing the plane into $f(n)$ distinct regions. Find a recurrence relation for $f(n)$ and solve it for $f(n)$. Then find the value of $f(9)$.

3.3 Solve the Tower of Hanoi Problem.

3.4 In climbing up a staircase, an ordinary step covers at least one stair and at most two stairs. If $f(n)$ is the number of ways of climbing up a staircase (making only ordinary steps) with n stairs, find a recurrence relation for $f(n)$.

3.5 Let S be the set of all binary words of length n such that two zeros do not appear consecutively in any word in the set. Find a recurrence relation for the number of elements in S.

3.9 Let $f(n)$ be the number of elements in X that is the set of all n-symbol words formed from the symbols A, B and C such that no word has a pair of consecutive A's. Find a recurrence relation for $f(n)$.

3.11 Solve $f(n+3) = 6f(n+2) - 11f(n+1) + 6f(n)$, where $f(0) = 3, f(1) = 6$ and $f(2) = 14$.

3.13 Solve $f(n+3) = 3f(n+2) + 4f(n+1) - 12f(n)$, where $f(0) = 0, f(1) = -11$ and $f(2) = -15$.

3.14 The roots of the characteristic equation of a linear homogeneous recurrence relation with constant coefficients are 1,2,2 and 3. Write down the relation and its general solution.

3.17 Solve the following inhomogeneous recurrence relations involving $f(n) : f(n) - 4f(n-1) + 4f(n-2) = h(n)$, where (a) $h(n) = 1$ (b) $h(n) = n$ (c) $h(n) = 3^n$ (d) $h(n) = 2^n$ (e) $h(n) = 1 + n + 2^n + 3^n$.

3.19 Solve $f(n) = 4f(n-1) + 5 \cdot 3^n$.

3.20 Solve $f(n) = 4f(n-1) + 5 \cdot 4^n$.

3.21 Solve $f(n) = f(n-1) + 2f(n-2) + 4 \cdot 3^n$ with the initial conditions $f(0) = 11$, $f(1) = 28$.

3.22 Solve $f(n) = 4f(n-1) - 4f(n-2) + 2^n$.

3.25 If the ordinary generating function of a recurrence relation involving $f(n)$ is

$$g(x) = \frac{2}{(1-x)(1-2x)},$$

find $f(n)$.

3.26 Solve the recurrence relation $f(n) = f(n-2) + 4n$, with $f(1) = 2, f(0) = 3$, by using the appropriate ordinary generating function.

3.31 Find the ordinary generating function for the recurrence relation $f(n+1) = af(n) + b^n$ with the initial condition $f(0) = c$, where a, b, c are constants.