Theory to learn for third exam.

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Two of the following questions will be on the exam:

1. Use generating functions to provide formulas for the r-combinations and the r-collections of a set of n-elements. Explain in detail.

   Hint: Done in detail in class.

2. Define carefully the extended binomial coefficient $\binom{u}{k}$, for $u \in \mathbb{R}$ and k a nonnegative integer. Then write down the statement of the extended binomial theorem. Finally, show that, if n is a positive integer, then $\binom{-n}{k} = \binom{n+k-1}{k}$.

   Hint: Done in detail in class.

3. Show that $(1-4x)^{-\frac{1}{2}} = \sum_{k=0}^{\infty} \binom{2k}{k} x^k$.

   Hint: Done in detail in class.

4. Provide the formula for the infinite sum of the terms of the geometric series $a, ar, ar^2, ar^3, \ldots$. Also provide the formula for the sum of the first n terms of the same series. Use your formulas to prove the relationship

   $$(1 + x + x^2 + x^3 + \ldots + x^{n-1}) = (1 - x^n)(1 + x + x^2 + x^3 + \ldots).$$

   Hint: Done in detail in class. (It is my opinion that you should also **know how to derive the first two formulas** but you do not have to show this in your answers.)

5. Prove that if $A(x) = \sum_{k=0}^{\infty} a_k$ and $B(x) = \sum_{k=0}^{\infty} b_k$, then

   (a) $A(x)B(x) = \sum_{k=0}^{\infty} (\sum_{i=0}^{k} a_i b_{k-i}) x^k$

   (b) $xA'(x) = \sum_{k=0}^{\infty} k a_k x^k$.

   Hint: Done in detail in class.

6. Show that $(1-2x)^{-\frac{3}{2}}$ is the exponential generating function of the sequence $1, 1 \cdot 3, 1 \cdot 3 \cdot 5, 1 \cdot 3 \cdot 5 \cdot 7, \ldots$.

   Hint: Done in detail in class.

7. Use exponential generating functions to show that the number of the r-digit quaternary sequences in which each of the digits $1, 2, 3$ appears at least once is $4^r - 3 \cdot 3^r + 3 \cdot 2^r - 1$.

   Hint: Done in detail in class.

8. Find the exponential enumerator for the number of ways to choose r or less out of r distinct objects and distribute them in n distinct cells with the objects in the same cell ordered.

   Hint: Done in detail in class.

In addition, **two problems** out of your **third homework set** plus one **wild-card problem** will be chosen to complete the five questions on your exam.