

EXAM 1 - MATH 112

DATE: Friday, February 2

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the following limits:

(a) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{3x - x^2 - 2}$ (2.5 point)

(b) $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$ (2.5 points)

2. Consider the function $f(x) = \begin{cases} x^2 - 1, & \text{if } x < 2 \\ 7, & \text{if } x = 2 \\ -x + 5, & \text{if } x > 2 \end{cases}$

(a) Roughly sketch the graph of $y = f(x)$. (2 points)

(b) Find the $\lim_{x \rightarrow 2} f(x)$, if it exists. (2 points)

(c) Is f continuous at $x = 2$? (1 point)

3. Use the **limit definition of the derivative** to compute the equation of the tangent line to the graph of $f(x) = \sqrt{x+2}$ at the point $(7, 3)$. (5 points)

4. (a) Find an equation for the tangent line to the graph of $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$ at $x = 1$. (3 points)

(b) At which points does the graph of the function $f(x) = x^3 + x$ have a horizontal tangent line? (2 points)

5. Suppose that the demand function for a certain product is $p(x) = 5 + \frac{10}{\sqrt{x}}$.

(a) Find the revenue from the sales of 100 units of the product. (2 points)

(b) Find the marginal revenue when 100 units of the product are sold. (3 points)

6. (a) Find the derivative of the function $f(x) = \frac{x^2 + 2}{x + 4}$. (3 points)

(b) Find the slope of the tangent line to the graph of $y = f(x)$ at $x = 2$. (1 points)

(c) Find the equation to the tangent of $y = f(x)$ at $x = 2$. (1 point)