EXAM 1 - MATH 151

DATE: Friday, September 22

INSTRUCTOR: George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Starting from the graph of $f(x) = \sqrt{x}$ give the step-by-step transformations that need to be followed to graph the function $g(x) = \sqrt{4-2x} - 6$. (You do not need to do the actual graphing.) (3 points)

   (b) Which function $g(x)$ has the same graph as the graph of $f(x) = \frac{1}{x}$ after it is shifted two points down, reflected with respect to the x-axis, stretched vertically by a factor of 3 and moved right by 5 points (in the given order)? Present carefully all intermediate steps. (2 points)

2. Suppose that $f(x) = \frac{3}{x-1}$ and $g(x) = \sqrt{x+3}$.

   (a) Find the domains $\text{Dom}(f)$ and $\text{Dom}(g)$. (1 point)

   (b) Find a formula for $(g \circ f)(x)$ and simplify. (2 points)

   (c) Find the domain of $g \circ f$. (2 points)

3. (a) Graph the piece-wise defined function $f(x) = \begin{cases} (x+1)^3, & \text{if } x < -1 \\ -x^2 + 1, & \text{if } -1 \leq x < 2 \\ x - 3, & \text{if } x \geq 2 \end{cases}$ (2 points)
   Find the limits $\lim_{x \to -1} f(x)$ and $\lim_{x \to 2} f(x)$ (1 point).

   (b) Suppose that $\frac{1}{2}x + 1 \leq f(x) \leq x^2 - 4x + 6$ for all x. Find the $\lim_{x \to 2} f(x)$. Justify your answer. (2 points)

4. Find the following limits:

   (a) $\lim_{x \to 1} \frac{x^2+4x-5}{-x^2+4x-3}$ (2 points)

   (b) $\lim_{x \to 11} \frac{11-x}{\sqrt{x-2}-3}$ (2 points)

   (c) $\lim_{t \to 0} (\frac{1}{t} + \frac{1}{t^2-t})$ (1 point)

5. Find the following trigonometric limits:

   (a) $\lim_{\theta \to 0} \frac{\sin 3\theta}{\sin 7\theta}$ (2 point)

   (b) $\lim_{t \to 0} \frac{5t^2}{\sin^2 2t}$ (3 points)

6. (a) Complete the formal definition: "A function f(x) is continuous at $x = a$ if ...". (2 points)

   (b) Consider the function $g(x) = \begin{cases} \frac{x^2-x-6}{x^2+x-2}, & \text{if } x \neq -2, 1 \\ a, & \text{if } x = -2 \\ b, & \text{if } x = 1 \end{cases}$ Find the value of a (if any) so that g be continuous at $x = -2$ (2 points). Find the value of b (if any) so that g be continuous at $x = 1$ (1 point).