## EXAM 3 - MATH 151

DATE: Monday, March 26

**INSTRUCTOR:** George Voutsadakis

Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

## GOOD LUCK!!

- (a) At which point(s) (x, y) on the curve y = xe<sup>-x<sup>2</sup></sup> is the tangent horizontal? (3 points)
  (b) Find the derivative of f(x) = (sin x)<sup>ln x</sup>. (2 points)
- 2. (a) Write a formula for the derivative  $(f^{-1})'(a)$  in terms of the derivative of f. (1 point) (b) Show that  $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$ . (2 points)
  - (c) Compute the derivative of  $f(x) = \sqrt{1 x^2} \sin^{-1} x$ . (2 points)
- 3. (a) Use the definitions of the hyperbolic functions to show that  $\frac{1+\tanh x}{1-\tanh x} = e^{2x}$ . (2 points)
  - (b) Use the definitions of the hyperbolic functions to show that  $(\sinh x)' = \cosh x$ . (1 point)
  - (c) Compute the derivative of  $f(x) = \sinh \sqrt{1 + x^2}$ . (2 points)
- 4. Calculate the following limits:
  - (a)  $\lim_{x\to 0^+} \sin x \ln x$  (2 points)
  - (b)  $\lim_{x\to 0} (1-2x)^{1/x}$  (3 points)
- 5. (a) State Rolle's Theorem and the Mean value Theorem. (2 points)
  - (b) Verify that  $f(x) = 3x^2 + 2x + 5$  satisfies all the hypotheses of the Mean Value Theorem on [-1, 1] and find all numbers c that satisfy its conclusion. (3 points)
- 6. Find the intervals of monotonicity, the relative extrema, the intervals of concavity and the inflection points of the function  $f(x) = 3x^5 5x^3 + 3$ . Summarize your findings in the form of a table. (5 points)