

# HOMEWORK 3 - MATH 151

DUE DATE: Thursday, February 15

INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Differentiate the following functions

$$(a) f(x) = x^3 - 4x + 6 \quad (b) f(x) = \frac{x^2 + 4x + 7}{\sqrt[3]{x}} \quad (c) f(x) = \sqrt[3]{x^2} + 2\sqrt{x^3}$$

2. Find the equations of the tangent line and the normal line to the curve  $f(x) = -6\cos x$  at  $(\frac{\pi}{3}, -3)$ .

3. Find the first and the second derivatives of the following functions

$$(a) f(x) = 2\cos x - 3\sin x \quad (b) g(x) = \sqrt{x} + 5\sin x$$

4. Find an equation for the normal line to the parabola  $y = x^2 - 5x + 4$  that is parallel to the line  $x - 3y = 5$ .

5. If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume  $V$  of water remaining in the tank after  $t$  minutes as  $V = 5000(1 - \frac{1}{40}t)^2, 0 \leq t \leq 40$ . Find the rate at which the water is draining from the tank after  $a$  minutes. At which time is the water draining out the fastest? How about the slowest?

6. Differentiate the following functions:

$$(a) f(x) = x^3 \sin x \quad (b) f(x) = (\frac{1}{x^2} - \frac{3}{x^4})(x + 5x^3) \quad (c) f(x) = \frac{x^2}{3x^2 - 2x + 1}$$

$$(d) f(x) = \frac{\sqrt{x} + 1}{\sqrt{x} - 1} \quad (e) f(x) = \frac{1 + \cos x}{x + \sin x}$$

7. If  $f$  is a differentiable function find an expression for the derivatives of  $g$  and  $h$  in terms of the derivative of  $f$ :

$$(a) g(x) = x^2 f(x) \quad (b) h(x) = \frac{1 + x f(x)}{\sqrt{x}}.$$

8. Find the derivative of the following functions:

$$(a) f(x) = \frac{1}{(x^4 + 1)^7} \quad (b) f(x) = 7^3 + \cos^3 x \quad (c) f(x) = (x^2 + 1)\sqrt[3]{x^2 + 2} \quad (d) f(x) = \cos(x \sin x)$$

9. Find the derivative of the following functions:

$$(a) f(x) = \sqrt{\frac{x-1}{x+1}} \quad (b) f(x) = \sin \sqrt{1+x^2} \quad (c) f(x) = (1 + \cos^2 x)^9 \quad (d) f(x) = \cos(\cos(\cos x))$$

10. Find all points on the graph of  $f(x) = 2\sin x + \sin^2 x$  at which the tangent line is horizontal.