

# HOMEWORK 5 - MATH 151

DUE DATE: Monday, March 12

INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Compute the following derivatives:

$$(a) f(x) = \log_5(xe^x) \quad (b) f(x) = \frac{1 + \ln x}{1 - \ln x} \quad (c) f(x) = \ln(x^7 \sin^3 x)$$

$$(d) f(x) = \sqrt[3]{x} e^{9x} \quad (e) f(x) = \frac{1 - xe^x}{x + e^x}$$

2. Use logarithmic differentiation to compute the derivative of

$$(a) y = \sqrt{x}e^{x^2}(x^2 + 1)^{12} \quad (b) y = \sqrt[5]{\frac{x^3 + 1}{x^3 - 1}} \quad (c) y = (\sin x)^{\ln x}$$

3. (a) Find  $y'$  if  $e^{x^2y} = x + y$

(b) Find an equation of the tangent line to the curve  $xe^y + ye^x = 1$  at  $(0, 1)$ .

4. A sample of tritium-3 decayed to 94.5% of its original amount after a year. What is its half-time? How long would it take the sample to decay to 20% of its original amount?

5. A curve passes through the point  $(0, 5)$  and has the property that the slope of the curve at every point  $P$  is twice the  $y$ -coordinate of  $P$ . What is the equation of the curve?

6. (a) Prove that  $(\sec^{-1} x)' = \frac{1}{x\sqrt{x^2-1}}$ .

(b) If  $g(x) = x \sin^{-1}(x/4) + \sqrt{16 - x^2}$ , find  $g'(2)$ .

7. Find the derivatives of the following functions

$$(a) f(x) = \sqrt{1 - x^2} \arcsin x \quad (b) f(x) = x \ln(\arctan x) \quad (c) f(x) = x \cos^{-1} x - \sqrt{1 - x^2}$$

8. Prove the identities

$$(a) \cosh x - \sinh x = e^{-x} \quad (b) \cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$(c) \sinh 2x = 2 \sinh x \cosh x$$

9. Use the definitions of the hyperbolic functions to find each of the following limits:

$$(a) \lim_{x \rightarrow -\infty} \tanh x \quad (b) \lim_{x \rightarrow -\infty} \sinh x \quad (c) \lim_{x \rightarrow 0^-} \coth x$$

10. Find the derivatives

$$(a) f(x) = \sinh x \tanh x \quad (b) f(x) = \ln(\sinh x) \quad (c) f(x) = \sinh(\cosh x)$$