HOMEWORK 6 - MATH 151

DUE DATE: Friday, March 23

INSTRUCTOR: George Voutsadakis

Read each problem **very carefully** before starting to solve it. Four out of the ten problems will be chosen at random and graded. Each problem graded is worth 3 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Find the critical numbers of the following functions:
   (a) $f(x) = x^{4/5}(x-4)^2$
   (b) $f(x) = x \ln x$
   (c) $f(x) = xe^{2x}$
2. Find the absolute maximum and the absolute minimum values of f on the given interval:
   (a) $f(x) = x\sqrt{4-x^2}$ in $[-1, 2]$
   (b) $f(x) = x - \ln x$ in $[\frac{1}{2}, 2]$
3. Prove that the function $f(x) = x^{101} + x^{51} + x + 1$ has neither a local maximum nor a local minimum.
4. Verify that the function $f(x) = x^3 - 3x^2 + 2x + 5$ satisfies the three hypotheses of Rolle's Theorem on the interval $[0, 2]$. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.
5. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem:
   (a) $f(x) = x^3 + x - 1$ on $[0, 2]$
   (b) $f(x) = \frac{x}{x+2}$ on $[1, 4]$.
6. Show that the equation $1 + 2x + x^3 + 4x^5 = 0$ has exactly one real root.
7. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?
8. Use Theorem 5 on page 209 to prove the identity $2\sin^{-1} x = \cos^{-1}(1-2x^2), x \geq 0$.
9. Find the domains, the intercepts, the asymptotes, form the sign tables and then roughly sketch the graphs of the following functions:
   (a) $f(x) = x^4 - 4x - 1$
   (b) $f(x) = x^2e^x$
   (c) $f(x) = x \ln x$
10. Find the domains, the intercepts, the asymptotes, form the sign tables and then roughly sketch the graphs of the following functions:
    (a) $f(x) = \frac{x^2}{x^2-1}$
    (b) $f(x) = \frac{e^x}{1+e^x}$