PRACTICE EXAM 1 - MATH 151

DATE: Tuesday, January 30

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. Consider the function

$$f(x) = \begin{cases} 2x+2, & \text{if } x < 0\\ \cos x, & \text{if } 0 \le x < \frac{\pi}{2}\\ 1, & \text{if } x = \frac{\pi}{2}\\ \sqrt{x-\frac{\pi}{2}}, & \text{if } x > \frac{\pi}{2} \end{cases}$$

- (a) Sketch carefully the graph of f. (2 points)
- (b) Find $\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$, $\lim_{x\to \frac{\pi}{2}^-} f(x)$, $\lim_{x\to \frac{\pi}{2}^+} f(x)$. (2 points)
- (c) Use the previous part to find the limits $\lim_{x\to 0} f(x)$ and $\lim_{x\to \frac{\pi}{2}} f(x)$. (1 point)
- 2. Suppose that $f(x) = \frac{1}{x+3}$ and $g(x) = \sqrt{2-x}$.
 - (a) Find the domains Dom(f) and Dom(g). (1 point)
 - (b) Find a formula for $(g \circ f)(x)$ and simplify. (2 points)
 - (c) Find the domain of $g \circ f$. (2 points)
- 3. (a) Evaluate the following limits without using any substitutivity properties justifying each step: $\lim_{x\to 5} \frac{x^3 4x^2 + 1}{7x 9}$ and $\lim_{x\to 1} \sqrt[3]{\frac{x+1}{4x+12}}$. (3 points)
 - (b) Suppose that $15x 35 \le g(x) \le 2x^2 + 3x 17$ for all x. Find the $\lim_{x\to 3} g(x)$. Justify your answer. (2 points)
- 4. Find the following limits:
 - (a) $\lim_{x\to 2} \frac{x-2}{x^3-8}$ (2 points)
 - (b) $\lim_{h \to 0} \frac{\sqrt{3h+25}-5}{h}$ (2 points)
 - (c) $\lim_{t\to 0} \left(\frac{1}{t} \frac{1}{t^2 t}\right)$ (1 point)
- 5. Find the following trigonometric limits:
 - (a) $\lim_{\theta \to 0} \frac{\sin 5\theta}{2\theta}$ (1 point)
 - (b) $\lim_{t\to 0} \frac{\sin^2 3t}{2t^2}$ (2 points)
 - (c) $\lim_{\theta \to 0} \frac{\sin \theta}{\theta + \tan \theta}$ (2 points)
- 6. (a) Look again at the graph of f(x) that you sketched in the first problem. Then fill-in the following table with "Y" for yes and "N" for no's.

	f continuous from left at	f continuous from right at	f continuous at
x = 0			
$x = \frac{\pi}{2}$			

(b) Find the value of a so that the function $g(x) = \begin{cases} \frac{x^2+x-2}{x^2+4x-5}, & \text{if } x \neq 1 \\ a, & \text{if } x = 1 \end{cases}$ be continuous at x = 1.