PRACTICE EXAM 3 - MATH 151

DATE: Monday, March 26

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Read each problem very carefully before starting to solve it. Each question is worth 5 points. It is necessary to show your work. Correct answers without explanations are worth 0 points.

GOOD LUCK!!

1. (a) Find $(f^{-1})'(4)$ if $f(x) = 3 + x + e^x$. (2 points)
   (b) Find $f^{-1}(x)$ if $f(x) = \frac{1+e^x}{1-e^x}$. (2 points) Find also the domains of both f and f^{-1}. (1 point)

2. (a) Find an equation for the tangent line to the graph of $f(x) = (2+x)e^{-x}$ at $x = 0$. (3 points)
   (b) At which point (x, y) on the curve $y = [\ln(x+4)]^2$ is the tangent horizontal? (2 points)

3. Compute $\frac{dy}{dx}$.
   (a) $y = (\cos x)^x$ (2 points)
   (b) $y = x + \arctan y$ (2 points)
   (c) $xe^y = y - 1$ (1 point)

4. A bacteria culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour the population has increased to 360 cells.
   (a) Find the population of bacteria after t hours. (2 points)
   (b) Find the number of bacteria after 4 hours. (1 point)
   (c) Find the rate of growth after 4 hours. (1 point)
   (d) When will the population reach 10,000? (1 point)

5. Find the derivatives of the following functions:
   (a) $f(x) = \arctan(\arcsin\sqrt{x})$ (2 points)
   (b) $g(x) = \frac{1}{2}\tan^{-1}x + \frac{1}{4}\ln\frac{(x+1)^2}{x^2+1}$ (2 points)
   (c) $h(x) = \ln(\cosh 3x)$ (1 point)

6. Calculate the following limits:
   (a) $\lim_{x\to\infty}(1+\frac{4}{x})^x$ (1 point)
   (b) $\lim_{x\to 1^+}(\frac{x}{x-1} - \frac{1}{\ln x})$ (2 points)
   (c) $\lim_{x\to(\pi/2)^-}(\tan x)^{\cos x}$ (2 points)