EXAM 4 - MATH 111 Friday, November 19

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. The probability P (as a decimal) of no tsunami wave of height 15 feet or more striking Crescent City, CA, over a period of Y years decreases as the time interval increases. Increasing the time interval by 1 year decreases the probability by about 2%.

   (a) Explain why P is an exponential function of Y.

   (b) What is the decay factor for P?

   (c) The probability for certainty is 1. What is the initial value for P? Please explain.

   (d) Find a formula for P as a function of Y.

   (e) What is the probability of no tsunami waves 15 feet or higher striking Crescent City over a 10-year period? How about over a 100-year period?

2. A worker is reviewing his pay increases over the past several years. His hourly wage W in dollars that he earned as a function of time t since the beginning of 2000 is given by

Time t	1	2	3	4
Wage W	20.20	20.62	21.06	21.50

(a) Calculate the ratios of change to see whether the data in the table are exponential.

(b) What is the yearly growth rate for the data?

(c) Determine what the worker's hourly wage was in 2000.

(d) Find an exponential model for W versus t.

(e) What was the worker's yearly percentage raise?

(f) During the decade 2000-2009, prices increased by 35%. Did the workers wage increases keep up with inflation? Please, explain.

3. The table below gives the average number N of earthquakes of magnitude at least M that occur each year world-wide.

Magnitude M	6	6.1	6.6	7	7.3	8
Number N	95.8	77.8	27.6	12.1	6.5	1.5

(a) Gutenberg and Richter fitted these data with an exponential function. Find an appropriate exponential model for the data.

(b) How many earthquakes per year of magnitude at least 5.5 can be expected?

(c) How many earthquakes per year of magnitude 8.5 or greater does the model predict?

(d) What is the limiting value for N? What does it mean in practical terms? Explain.

(e) How does the number of earthquakes per year of magnitude M or greater compare with the number of earthquakes of magnitude $M + 1$ or greater?

4. For certain decisions the time it takes to respond is a logarithmic function of the number of choices faced. One model used is $R = 0.17 + 0.44 \log N$, where R is the reaction time in seconds and N is the number of choices.

   (a) Draw a graph of R versus N. Include values of N ranges from 1 to 10 choices.

   (b) Express using functional notation the reaction time if there are seven choices and then calculate that time.

   (c) If the reaction time can only be at most 0.5 second, how many choices can there be?

   (d) If the number of choices increases by a factor of 10, what happens to the reaction time?

   (e) Explain in practical terms what the concavity of the graph means.

5. Astronomers measure brightness of stars using both the *absolute magnitude*, a measure of the true brightness of the star, and the *apparent magnitude*, a measure of the brightness of the star as it appears from Earth. The difference between apparent and absolute magnitude should yield information about the distance to the star. The table below gives magnitude difference m and distance d, measured in light-years, for several stars:

Star	Magnitude Difference m	Distance d
Algol	2.56	105
Aldebaran	1.56	68
Capella	0.66	45
Canopus	2.38	98
Pollux	0.13	35
Regulus	2.05	84

(a) Plot $\ln d$ against m and determine whether it is reasonable to model distance as an exponential function of magnitude difference. Justify.

(b) Give an exponential model for the data using the logarithm as a link.

(c) If one star shows a magnitude difference 1 greater than the magnitude difference that a second star shows, how do their distances from Earth compare?

(d) Alphecca shows a magnitude difference of 1.83. How far is Alphecca from Earth?

(e) Alderamin is 52 light-years from Earth and has an apparent magnitude of 2.47. What is the absolute magnitude of Alderamin?