

YOUR NAME: \_\_\_\_\_

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Create a sign table for the first and second derivatives of the function

$$f(x) = 2x^4 - 8x^3 + 30$$

clearly showing intervals of monotonicity, relative extrema, intervals of concavity and inflection points of  $f$ . (You do not have to graph  $y = f(x)$ .)

2. Suppose that  $y = f(x)$  is continuous and differentiable everywhere and that we know the following data: (a)  $f(0) = 2$ ; (b)  $f'(x) > 0$  on  $(-\infty, -8)$  and  $(0, 8)$ ; (c)  $f'(x) < 0$  on  $(-8, 0)$  and  $(8, +\infty)$ ; (d)  $f''(x) < 0$  on  $(-\infty, -4)$  and  $(4, +\infty)$ ; (e)  $f''(x) > 0$  on  $(-4, 4)$ . **Neatly** summarize these data on a sign table and, then, sketch the graph of a function  $y = f(x)$  that satisfies conditions (a)-(e).

3. Find the absolute maximum and the absolute minimum values of  $f(x) = \frac{x}{x^2 + 1}$  on  $[-3, 3]$ .
4. A homeowner wants to build a garden surrounded by a fence along her driveway. The garden is going to be 5000 square feet and the fence along the driveway costs \$6 per foot while the fence for the other three sides costs only \$2 per foot. Find the dimensions of the garden that minimize the cost of the fencing.
5. Find the slope of the tangent line to the graph of the equation  $x^2 + y^2 = xy + 7$  at  $x = 3, y = 2$ .