

YOUR NAME: \_\_\_\_\_

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Let  $f(x) = x^2 + 2x - 3$ .

(a) Find the location of the vertex:

(b) What is the opening direction (justify):

(c) Find the  $y$ -intercept:

(d) Find the  $x$ -intercepts:

(e) Sketch the graph of  $y = f(x)$ ;

2. Let  $f(x) = \frac{3x^3 - 3x^2 - 6x}{x^2 + x}$ . Calculate the following limits:

(a)  $\lim_{x \rightarrow -1} f(x) =$

(b)  $\lim_{x \rightarrow 0} f(x) =$

3. Consider the piece-wise function  $f(x) = \begin{cases} \frac{x}{x-2}, & \text{if } x < 1 \\ 3, & \text{if } x = 1 \\ \frac{x^2 + x - 2}{2x^2 - 3x + 1}, & \text{if } x > 1 \end{cases}$

(a) Find  $\lim_{x \rightarrow 1^-} f(x)$

(b) Find  $f(1) =$

(c) Find  $\lim_{x \rightarrow 1^+} f(x)$

(d) Is  $f$  continuous at  $x = 1$ ? If not, is it left continuous or right continuous? Justify your answer.

4. Use the **limit definition of the derivative** to compute  $f'(5)$ , if  $f(x) = 2x^2 - x$ ;

5. Suppose the temperature  $T$  in degrees at a certain point in space is given by  $T(t) = \sqrt{1-t}$ , where  $t$  is time in minutes. Find the instantaneous rate of change of the temperature, at  $t = \frac{3}{4}$  minutes.