Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Evaluate the improper integral $\int_0^\infty \dfrac{e^{-x}}{(1 + e^{-x})^3} dx$.

2. Solve the following differential equation subject to the given initial condition:

$$y' = y^2 e^x + y^2, \quad y(0) = 1.$$