EXAM 1 - MATH 251 YOUR NAME:

Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the area enclosed by the inner loop of the limaçon $r = 2\cos\theta - 1$.



- 2. Consider the equation $r = \frac{12}{4 3\cos\theta};$
 - (a) Write the equation in the standard form;
 - (b) Find the eccentricity e =
 - (c) Identify the conic:
 - (d) Give an equation for the directrix;
 - (e) Sketch the graph;

3. A plane flying east at 200 Km/h encounters a 40 Km/h wind blowing in the north east direction. Find the velocity (this is a vector) and the speed (this is a number) of the plane relative to the ground.

4. Find an equation of the sphere of radius R that is centered in the first octant and touches all three coordinate planes. Then provide an equation of the sphere that touches the *xy*-plane at the same point as the previous sphere and has half the radius of the previous one. Finally, compute the volume of the solid lying outside the small and inside the big sphere.



5. Consider the vectors $\mathbf{a} = \langle 1, 0, 1 \rangle$ and $\mathbf{b} = \langle 3, 2, 1 \rangle$.



(b) Find the component $\mathrm{comp}_{\mathbf{a}}\mathbf{b}$ and the projection $\mathbf{proj}_{\mathbf{a}}\mathbf{b}$ of \mathbf{b} along $\mathbf{a};$

(a) Find $\cos \theta$ where θ is the angle from **a** to **b**;

(c) Find a unit vector in the direction of **b**;

Formulas: Volume V of a sphere of radius r: $V = \frac{4}{3}\pi r^3$