EXAM 3 - MATH 310 Friday, November 7

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Show **in detail** that the functions $f(t) = \sin t$, $g(t) = \cos t$ and $h(t) = t$ are linearly independent over $\mathbb{R}$.

2. Use the method of undetermined coefficients to solve the third-order linear differential equation $y''' - 4y' = t^2 + e^t$.

3. Use variation of parameters to find a general solution of the third-order differential equation $y''' - y'' + y' - y = e^{-t}\sin t, 0 \le t < 2\pi$. You may **leave your answer in integral form**.

4. Find **from scratch** the Laplace transform $F(s)$ of the function $f(t) = e^{3t}\cosh 2t$.

5. Use the Laplace transform method to solve the initial value problem $y'' - 10y' + 9y = 5t$, with $y(0) = -1$ and $y'(0) = 2$.

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$, $\quad s > 0$
2. e^{at}	$\frac{1}{s-a}$, $\quad s > a$
3. t^n, $\quad n =$ positive integer	$\frac{n!}{s^{n+1}}$, $\quad s > 0$
4. t^p, $\quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$, $\quad s > 0$
5. $\sin at$	$\frac{a}{s^2+a^2}$, $\quad s > 0$
6. $\cos at$	$\frac{s}{s^2+a^2}$, $\quad s > 0$
7. $\sinh at$	$\frac{a}{s^2-a^2}$, $\quad s > \lvert a \rvert$
8. $\cosh at$	$\frac{s}{s^2-a^2}$, $\quad s > \lvert a \rvert$
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$, $\quad s > a$
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$, $\quad s > a$
11. $t^n e^{at}$, $\quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}$, $\quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}$, $\quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$, $\quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $(-t)^n f(t)$	$F^{(n)}(s)$