EXAM 4 - MATH 310 Monday, November 24

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. We know that $\mathcal{L}\{t^6\} = \dfrac{6!}{s^7}$, for $s > 0$.

   (a) Use the definition of Laplace transform to write an integral equation equivalent to the given equation.

   (b) Use the equation of Part (a) to evaluate "from scratch" the Laplace transform $\mathcal{L}\{t^7\}$.

2. Find the Laplace tranform of the function $f(t) = \begin{cases} 0, & \text{if } t < 3 \\ (t-3)^5, & \text{if } t \geq 3 \end{cases}$.

3. Suppose $F(s) = \dfrac{3s^2 + 2s - 17}{s^3 - 8s^2 + 17s}$. Find $\mathcal{L}^{-1}\{F(s)\}$.

4. Solve the initial value problem

$$y'' + 4y = \delta(t - \pi), \quad y(0) = 2, y'(0) = 0.$$

5. Express the solution of

$$y'' + 3y' + 2y = \cos \omega t, \quad y(0) = 1, y'(0) = 0$$

in terms of a convolution integral.

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. t^n, $n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > \lvert a \rvert$
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > \lvert a \rvert$
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
11. $t^n e^{at}$, $n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t) f(t-c)$	$e^{-cs} F(s)$
14. $e^{ct} f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau) g(\tau)\, d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $(-t)^n f(t)$	$F^{(n)}(s)$