QUIZ 9 - MATH 310 — Friday, November 14

YOUR NAME:________________________ — George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Graph and express the function $g(t) = \begin{cases} 0, & \text{if } 0 \leq t < \pi \text{ or } t \geq 3\pi \\ 1, & \text{if } \pi \leq t < 3\pi \end{cases}$ in terms of Heavyside functions.

2. Decompose the fraction $\dfrac{1}{s(s^2+4)}$ into partial fractions.

3. Find the inverse Laplace transforms of the following functions:

$$e^{-\pi s}\frac{1}{s} =$$

$$e^{-3\pi s}\frac{1}{s} =$$

$$e^{-\pi s}\frac{s}{s^2+4} =$$

$$e^{-3\pi s}\frac{s}{s^2+4} =$$

4. Use Laplace transforms to solve the initial value problem

$$y'' + 4y = g(t), \quad y(0) = 0,\ y'(0) = 0,\ g(t) = \begin{cases} 0, & \text{if } 0 \le t < \pi \text{ or } t \ge 3\pi \\ 1, & \text{if } \pi \le t < 3\pi \end{cases} .$$

(**Hint**: Take advantage of Problems 1-3 in your solution!!)

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > \lvert a \rvert$
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > \lvert a \rvert$
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t) f(t-c)$	$e^{-cs} F(s)$
14. $e^{ct} f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau) g(\tau)\, d\tau$	$F(s) G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $(-t)^n f(t)$	$F^{(n)}(s)$