

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Use the combined sign table for the first and the second derivatives to find the interval of monotonicity, relative extrema, intervals of concavity, inflection points and to roughly sketch the graph of $f(x) = x^4 + 4x^3 + 15$. Please, label all points in the graph clearly!

2. Suppose it costs \$200 to produce each motorbike and that the fixed costs are \$1500 per day. The price function is $p(x) = 600 - 5x$, where p is price at which exactly x bikes will be sold. How many motorbikes should be produced and how should they be priced to maximize profit?

(a) Cost function $C(x) =$

(b) Revenue function $R(x) =$

(c) Profit function $P(x) =$

(d) Maximize Profit to answer questions.

3. Solve the logarithmic equation

$$\ln(3x + 8) = \ln(2x + 2) + \ln(x - 2).$$

4. Find an equation for the tangent line to the graph of $f(x) = x^3 \ln x$ at $x = e$.

5. The number (in thousands) x of bacteria of type X and the number (in thousands) y of bacteria of type Y that can coexist in a cubic centimeter of nutrient are related by the equation $xy^2 + 2x^2y = 64$. Find $\frac{dy}{dx}$ at $x = 2$ and interpret your answer.