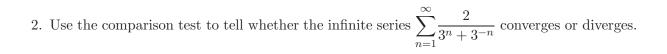Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Provide all details.

   (a) Find the limit of the sequence $a_n = \sqrt{n}\ln\left(1 + \frac{1}{n}\right)$.

   (b) Find the sum of the infinite series $\dfrac{2^3}{7} + \dfrac{2^4}{7^2} + \dfrac{2^5}{7^3} + \cdots$.

2. Use the comparison test to tell whether the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{3^n + 3^{-n}}$ converges or diverges.

3. Use the root test to tell whether the series $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2016n}{2015n + 2017} \right)^n$ converges or diverges.

4. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3n}(x-7)^n$.

5. Use your knowledge of the power series for the function $\dfrac{1}{1-x}$ and its radius of convergence to provide a power series for the function $f(x) = \dfrac{x}{(1-x^3)^2}$ in the sigma notation and state its radius of convergence. (**Hint**: Differentiation is involved.)