

YOUR NAME: _____

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Compute the integral

$$\int \tan^3 x \sec^2 x dx.$$

2. Use the method of trigonometric substitution to evaluate the integral

$$\int x^3 \sqrt{9 - x^2} dx.$$

3. Compute the integral

$$\int \tan^2 x \sec^3 x dx.$$

We compute first $\int \sec^5 x dx$.

$$\begin{aligned} \int \sec^5 x dx &= \int \sec^3 x \sec^2 x dx \\ &= \int \sec^3 x (\tan x)' dx \\ &= \tan x \sec^3 x - \int \tan x (\sec^3 x)' dx \\ &= \tan x \sec^3 x - \int \tan x 3 \sec^2 x \tan x \sec x dx \\ &= \tan x \sec^3 x - \int 3 \tan^2 x \sec^3 x dx. \end{aligned}$$

Next we compute $\int \sec^3 x dx$.

$$\begin{aligned} \int \sec^3 x dx &= \int \sec x \sec^2 x dx \\ &= \int \sec x (\tan x)' dx \\ &= \tan x \sec x - \int \tan x (\sec x)' dx \\ &= \tan x \sec x - \int \tan x \tan x \sec x dx \\ &= \tan x \sec x - \int \tan^2 x \sec x dx \\ &= \tan x \sec x - \int (\sec^2 x - 1) \sec x dx \\ &= \tan x \sec x - \int \sec^3 x dx + \int \sec x dx \\ &= \tan x \sec x + \ln |\tan x + \sec x| - \int \sec^3 x dx. \end{aligned}$$

So we get

$$\int \sec^3 x dx = \frac{1}{2} \tan x \sec x + \frac{1}{2} \ln |\tan x + \sec x| + C.$$

Finally, we calculate:

$$\begin{aligned} \int \tan^2 x \sec^3 x dx &= \int (\sec^2 x - 1) \sec^3 x dx \\ &= \int \sec^5 x dx - \int \sec^3 x dx \\ &= \tan x \sec^3 x - \int 3 \tan^2 x \sec^3 x dx \\ &\quad - \frac{1}{2} \tan x \sec x - \frac{1}{2} \ln |\tan x + \sec x|. \end{aligned}$$

Therefore

$$4 \int \tan^2 x \sec^3 x dx = \tan x \sec^3 x - \frac{1}{2} \tan x \sec x - \frac{1}{2} \ln |\tan x + \sec x|$$

and, finally,

$$\int \tan^2 x \sec^3 x dx = \frac{1}{4} \tan x \sec^3 x - \frac{1}{8} \tan x \sec x - \frac{1}{8} \ln |\tan x + \sec x| + C.$$