EXAM 1 - MATH 305 Friday, September 22

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Solve the system of linear equations by reducing the augmented matrix in **reduced** echelon form:
$$\left\{\begin{array}{rcl} x_1+2x_2+2x_3 &=& 1\\ 2x_1+5x_2+3x_3 &=& 4\\ 2x_2-x_3 &=& 1 \end{array}\right\}.$$

2. For which values of the real number c is the set $\left\{\begin{bmatrix}1\\3\\4\end{bmatrix},\begin{bmatrix}1\\4\\3\end{bmatrix},\begin{bmatrix}1\\1\\c\end{bmatrix}\right\}$ of vectors in $\mathbb{R}^3$ lineraly independent?

3. Suppose that $A = \begin{bmatrix} 1 & -2 & 1 & -7 & 1 \\ -1 & 2 & 1 & -5 & 1 \\ 2 & -4 & 2 & -14 & 4 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 18 \\ -6 \\ 50 \end{bmatrix}$. Solve the matrix equation $A\boldsymbol{x} = \boldsymbol{b}$ and provide its set of solutions in parametric vector form.

4. Consider the matrix $A = \begin{bmatrix} 1 & 0 & 3 & 5 \\ 1 & 2 & 5 & 9 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ and the linear transformations $T : \mathbb{R}^4 \to \mathbb{R}^3$ defined by $T(\boldsymbol{x}) = A\boldsymbol{x}$.

   (a) Is the vector $\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in the range of T? If yes, exhibit a vector whose image under T is $\boldsymbol{b}$.

   (b) Express all vectors $\boldsymbol{x}$ that are mapped into $\mathbf{0}$ by T in parametric vector form.

5. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation that maps $\boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\boldsymbol{e}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} 7 \\ -3 \end{bmatrix}$.

   (a) Find the images of the vectors $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ under T.

   (b) Find a matrix A so that $T(\boldsymbol{x}) = A\boldsymbol{x}$, for all $\boldsymbol{x}$ in $\mathbb{R}^2$.