EXAM 4 - MATH 305 Friday, December 8

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the eigenvalues and bases for the corresponding eigenspaces of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.

2. Diagonalize the matrix $A = \begin{bmatrix} 1 & -2 \\ -7 & 6 \end{bmatrix}$, i.e., find an invertible matrix P and a diagonal matrix D, such that $P^{-1}AP = D$.

3. Find a matrix X, such that $X^3 = A$, where A is the matrix of Problem 2.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(\boldsymbol{x}) = A\boldsymbol{x}$, where $A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}$.

If $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$, find $[T]_{\mathcal{B}}$ $(= {}_{\mathcal{B}}[T]_{\mathcal{B}})$.

5. Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, defined by $T(\boldsymbol{x}) = A\boldsymbol{x}$, where $A = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix}$. Find the following:

   (a) A basis $\mathcal{B}$ of $\mathbb{R}^2$, such that $[T]_{\mathcal{B}}$ is diagonal.

   (b) $[T]_{\mathcal{B}}$ where $\mathcal{B}$ is the basis you found in Part (a).