Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

- 1. Consider the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.
 - (a) Find the characteristic polynomial $p(\lambda)$ of A.

(b) Find the characteristic equation of A.

(c) Find the eigenvalues of A with their (algebraic) multiplicities.

- 2. Consider again $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.
 - (a) Find for each eigenvalue λ of A a basis for the corresponding eigenspace.

(b) If A is diagonalizable, find an invertible matrix P and a diagonal matrix D, such that $P^{-1}AP=D$.