

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Check whether the transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} 2x_1 - 3x_2 \\ x_1 + 4 \\ 5x_2 \end{bmatrix}$$

is linear or not. (If yes, then prove. If not, then provide a counterexample justifying why not.)

2. Consider the transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} x_1 - x_2 + x_3 \\ x_1 + 2x_2 - x_3 \end{bmatrix}.$$

- (a) Is T one-to-one? Justify your answer.

- (b) Is T onto? Justify your answer.

3. Consider the transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that rotates a vector by $\frac{\pi}{4}$ radians clockwise and doubles its length. Find the standard matrix of T .