

YOUR NAME: \_\_\_\_\_

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Let  $M_2$  be the vector space of all  $2 \times 2$  square matrices. Consider the transformation  $T :$

$$M_2 \rightarrow M_2, \text{ defined by } T(A) = A + A^T, \text{ for all } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2.$$

- (a) Show that  $T$  is a **linear** transformation.

- (b) Let  $B \in M_2$  be such that  $B^T = B$ . Find an  $A \in M_2$ , such that  $T(A) = B$ .

- (c) Describe precisely the kernel of  $T$ .

2. Consider the matrix  $A = \begin{bmatrix} 1 & 0 & -5 & 4 \\ -2 & 1 & 6 & -2 \\ 0 & 2 & -8 & 12 \end{bmatrix}$ .

**Hint:** Parts (b) and (c) rely on Part (a).

(a) Reduced  $A$  to row reduced echelon form.

(a) Find a basis for  $\text{Nul}A$ .

(c) Find a basis for  $\text{Col}A$ .